

# MATH

## EXPRESSIONS\*

### MULTIPLICATION & DIVISION

### "Basic Facts"

#### Strategies

- o count by n for multiplication and division
- o square numbers (the product of a whole # and itself)  
(ex.  $4 \times 4 = 16$ , 16 is a square #)
- o commutativity  
 $8 \times 4 = 4 \times 8$
- o inverse operations – think to unmultiply or to find a missing factor  
 $8 \times 4 = 32$  so  
 $32 \div 4 = 8$   
 $32 \div 8 = 4$

#### Derived Facts—using facts you know to figure out those you don't

- o Start with a known, count by n and then add on  
 $4 \times 6 = 4 \times 5$  plus 4
- o Start with a known fact and double  
 $4 \times 8 = 4 \times 4$  then double
- o Combine 2 known facts  
 $4 \times 6 = 4 \times 4 + 4 \times 2$

#### Basic Multiplication & Division Facts are extended to larger numbers:

$$\begin{aligned} 4 \times 8 &= 32 \\ 4 \times 8 \times 10 &= 320 \text{ so} \\ 40 \times 8 &= 320 \\ 4 \times 80 &= 320 \\ 4 \times 8 \times 100 &= 3200 \\ 40 \times 80 &= 3200 \text{ so} \\ 4 \times 800 &= 3200 \\ 400 \times 8 &= 3200 \end{aligned}$$

Multiplication problems can be shown as  $4 \times 3$  or  $4 \cdot 3$  or  $4 * 3$ .  
Division problems can be shown as  $50 \div 5$  or  $\frac{50}{5}$  or  $50/5$  or  $5 \overline{)50}$ .

## Multiplication & Division Situations: Problem Types

#### Equal Groups – unknown total

I have 3 bowls. There are 2 oranges in each bowl. How many oranges in all?

#### Equal Groups – unknown number of groups (measurement division)

(think: breaking off a chunk or repeated subtraction)

I have 6 oranges. I give 2 to each person who comes in. How many people get oranges?

#### Equal Groups – unknown group size (fraction division)

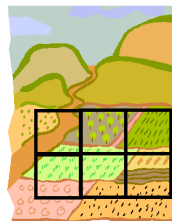
(think: fair share or dealing)

I have 6 oranges. I share them equally between Tom, Sue, and Maria. How many oranges does each get?

#### Array

(think: things in natural rows & columns)

In my album I have 3 rows of pictures with 2 pictures in each row. How many pictures?



#### Area

(think: clear grid overlaying a region)

Our family owns a piece of land that is 3 miles wide and 2 miles long. How much land do we own?

#### Combinations

(think: total # of possible outcomes)

For your ice cream sundae you may choose 1 of 3 ice cream flavors and 1 of 2 toppings. How many different sundaes could you make?

#### Comparison

(think: a comparison using multiplication)

I have a dog who is 2 feet tall. My big brother is three times as tall as my dog. How tall is he?

## Algorithms

### Multiplication of Whole Numbers:

#### Rectangle Sections/ Partial Products

$$23 \times 36 = 828$$

	30	6	
20	$20 \times 30 = 600$	$20 \times 6 = 120$	
3	$3 \times 30 = 90$	$3 \times 6 = 18$	

$$\begin{array}{r} 600 \\ 120 \\ 90 \\ + 18 \\ \hline 828 \end{array}$$

#### Expanded Notation

$$\begin{aligned} 36 &= 30 + 6 \\ \times 23 &= 20 + 3 \\ \hline 20 \times 30 &= 600 \\ 20 \times 6 &= 120 \\ 3 \times 30 &= 90 \\ 3 \times 6 &= 18 \\ \hline &= 828 \end{aligned}$$

#### Algebraic Notation

$$\begin{aligned} 23 \times 36 &= (20 + 3) \times (30 + 6) \\ &= 600 + 120 + 90 + 18 \\ &= 828 \end{aligned}$$

#### Shortcut Notation (common U.S.)

$$\begin{array}{r} 1 \\ 1 \\ 36 \\ \times 23 \\ \hline 108 \\ 72 \\ \hline 828 \end{array}$$

### Division of Whole Numbers:

#### Partial Quotients (at least or the big 7)

$$\begin{array}{r} 19R3 \\ 12 \overline{)231} \\ - 120 \\ \hline 111 \\ - 60 \\ \hline 51 \\ - 48 \\ \hline 3 \end{array}$$

#### Expanded Notation

$$\begin{array}{r} 6 \\ 40 \\ 500 \\ 7 \overline{)3,822} \\ - 3,500 \\ \hline 322 \\ 280 \\ \hline 42 \\ - 42 \\ \hline 0 \end{array}$$

#### Digit by Digit (traditional)

$$\begin{array}{r} 546 \\ 7 \overline{)3,822} \\ - 35 \\ \hline 322 \\ 28 \\ \hline 42 \\ - 42 \\ \hline 0 \end{array}$$

#### Vocabulary Review

"Product" is the answer to a multiplication problem.  
"Quotient" is the answer to a division problem.

# MATH

## EXPRESSIONS\*

### ADDITION & SUBTRACTION

### "Basic Facts"

#### General Strategies

- o Counting on...
  - o Make a 10
 
$$8 + 5 = 8 + 2 + 3 = 13$$
  - o Partners & Switch Partners
 
$$6 = 5 + 1 = 4 + 2 = 3 + 3 \text{ etc.}$$

$$6 = 5 + 1 = 1 + 5$$

#### Specific Strategies

- o Doubles
 
$$4 + 4 = 8$$
- o Doubles +/- 1
 
$$7 + 6 = 6 + 6 + 1 = 13$$

$$7 + 6 = 7 + 7 - 1 = 13$$
- o Teens as 10 plus n
 
$$13 = 10 + 3$$

#### Basic Addition & Subtraction Facts are extended to larger numbers:

$$6 + 7 = 13$$

$$60 + 70 = 130$$

$$600 + 700 = 1300$$

## Addition & Subtraction Situations: Problem Types

#### Change Plus/Join

Chris has 6 books on animals. Her parents give her 7 more animal books. How many does she have in total?

#### Change Minus/Separate

Mike has 13 tickets to the zoo and he gives 6 of them to his cousins. How many does he have left?

#### Comparison

Carlos has 7 beautiful sea shells. Lee has 13 beautiful shells. How many more does Lee have than Carlos?

#### Collection: Part-Part-Whole

In her bedroom, Lynn has a shelf full of stuffed animals. Six are red and 7 are purple. How many does she have in all?

## Parent Math Expressions Website

[www.eduplace.com/math/mthexp/](http://www.eduplace.com/math/mthexp/)  
(includes eglossary, emanipulatives, egames)

## Additional NCTM Websites

<http://figurethis.org/>  
<http://illuminations.nctm.org/>  
(activities, lessons, games)

## Algorithms

### Addition of Whole Numbers

#### Show All Totals/Partial Sums

$$\begin{array}{r} 237 \\ + 59 \\ \hline 200 \\ 80 \\ + 16 \\ \hline 296 \end{array} \qquad \begin{array}{r} 237 \\ + 59 \\ \hline 16 \\ 80 \\ + 200 \\ \hline 296 \end{array}$$

#### New Groups Below

$$\begin{array}{r} 237 \\ + 59 \\ \hline 1 \\ \hline 296 \end{array}$$

#### New Groups Above (common U.S.)

$$\begin{array}{r} 1 \\ 237 \\ + 59 \\ \hline 296 \end{array}$$

### Subtraction of Whole Numbers

#### Expanded Method

$$\begin{array}{r} 120 \\ 130 \quad 16 \\ 136 = 100 + 30 + 6 \\ - 47 = - \quad 40 + 7 \\ \hline 80 + 9 = 89 \end{array}$$

#### Ungroup First then Subtract Everywhere Method/ Trades First

$$\begin{array}{r} 12 \\ 0 \neq 16 \\ \hline 136 \\ - 47 \\ \hline 89 \end{array} \qquad \begin{array}{r} 12 \\ 0 \neq 16 \\ \hline 136 \\ - 47 \\ \hline 89 \end{array}$$

left to right or right to left

#### Alternating Ungroup & Subtract Method (common U.S.)

$$\begin{array}{r} 2 \\ 1 \neq 16 \\ \hline 136 \\ - 47 \\ \hline 9 \end{array} \qquad \begin{array}{r} 12 \\ \neq \neq 16 \\ \hline 136 \\ - 47 \\ \hline 89 \end{array}$$

step one                      step two

#### = Sign Review

= can also be read as "the same value as" or "is" or "is the same as"

\* The equals sign does not mean "the answer comes next."

#### Vocabulary Review

"Sum" is the answer to an addition problem.

"Difference" is the answer to a subtraction problem.

\*Developed from Math Expressions (Houghton Mifflin Harcourt, 2009)

\*Based on the work of Edmonds School District #15, Lynnwood, WA

